# A hybrid bio-inspired learning algorithm for image segmentation using multilevel thresholding 

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Received: 26 September 2015 / Revised: 10 August 2016 / Accepted: 22 August 2016
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#### Abstract

In the field of image analysis, segmentation is one of the most important preprocessing steps. One way to achieve segmentation is the use of threshold selection, where each pixel that belongs to a determined class, based on the mutual visual characteristics, is labeled according to the selected threshold. In this work, a combination of two pioneer methods, namely Otsu and Kapur, are investigated to solve the threshold selection problem. Optimum parameters of these objective functions are calculated using Bacterial Foraging (BF) optimization algorithm, for its accuracy, and Harmony Search (HS), for its speed. However, the biggest problem of soft computing family algorithms is catching into a local optimum. To resolve this critical issue, we investigate the power of Learning Automata (LA) which works as a controller to make switching between these two optimization methods. LA is a heuristic method which can solve complex optimization problems with interesting results in parameter estimation. Despite other techniques commonly seek through the parameter map, LA explores in the probability space, providing appropriate convergence properties and robustness. The proposed method is


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tested on benchmark images and shows fast convergence avoiding the typical sensitivity to initial conditions such as the Expectation-Maximization (EM) algorithm or the complex, and time-consuming computations which are commonly found in gradient methods. Experimental results demonstrate the algorithm's ability to perform automatic multithreshold selection and show interesting advantages as it is compared to other algorithms solving the same task.

Keywords Multilevel thresholding • Image segmentation • Hybrid optimization • Kapur function • Otsu function

## 1 Introduction

Image segmentation facilitates the separation of spatial-spectral attributes contained in images into their individual constituents; a task that is accomplished quite comfortably by our visual system and cortical mechanisms. However, mimicking this capability of human observers in an artificial environment has been found to be an extremely challenging problem. Formally, image segmentation is defined as the process of partitioning or segregating an image into regions (also called as clusters or groups), manifesting homogeneous or nearly homogeneous attributes such as color, texture, gradient and spatial attributes about location. Fundamentally, a segmentation algorithm for an image is said to be "complete" when it provides a unique region or label assignment for every pixel, such that all pixels in a segmented region satisfy certain criteria while the same principles are not universally satisfied for pixels from disjoint regions.

In the context of imagery, segmentation is often viewed as an ill-defined problem with no perfect solution but multiple acceptable solutions due to its subjective nature [23]. The subjectivity of segmentation has been extensively substantiated in experiments conducted at the University of California at Berkeley [14] to develop an evaluation benchmark, where a database of manually generated segmentations of images with natural content was developed using multiple human observers. In Fig. 1a, two images from the database as mentioned earlier are displayed. Additionally, several manually segmented ground truths with region boundaries superimposed (in green) on the original image are shown in Fig. 1b to f. Analysis of the obtained ground truth results by Martin et al. [14] divulged two imperative aspects: (I) an arbitrary image may have a unique suitable segmentation outcome while others possess multiple acceptable solutions, and (II) the variability of inadequate solutions is primarily due to the differences in the level of attention (or granularity) and the degree of detail from one human observer to another, as seen in Fig. 1. Consequently, most present day algorithms for segmentation aim to provide acceptable outcomes rather than a "gold standard" solution.

The most segmentation modus operandi can be viewed as being either spatially blind or spatially guided. Spatially blind approaches perform segmentation in certain attribute/feature spaces, predominantly related to intensity. Popular segmentation techniques that fall within the notion of being spatially blind involve clustering [4,25] and histogram thresholding [20]. In contrast to spatially blind methods, spatially guided approaches, as the name suggests, are guided by spatial relationships of pixels for segmentation. Their primary objective is to form pixel groupings that are compact or


Fig. 1 Berkeley segmentation benchmark [14] (a) original images, and (b) to (f) region boundaries of multiple manually generated segmentations overlaid on the images
homogeneous from a spatial standpoint, irrespective of their relationships in specific feature spaces. However, despite the development of many spatially guided techniques, the use of region and edge information explicitly or in an integrated framework remains widely-accepted alternatives for the formation of spatially compact regions. Segmentation approaches such as region-based [6], energy-based [18] and region and contour based $[8,15]$ fall within the notion of being spatially guided.

Histogram thresholding [20] is a spatially blind technique primarily based on the principle that segments of an image can be identified by delineating peaks, valleys, and/or shapes in its corresponding intensity histogram. Similar to clustering, histogram thresholding protocols require minimal effort to realize in comparison with most other segmentation algorithms and function without the need for any a priori information about the image being partitioned. Owed to its simplicity, intensity histogram thresholding initially gained popularity for segmenting gray-scale images which also has the capability of converting a gray-scale image into binary one.

In general, if the gray level histogram of the image is bi-modal, the image objects are clearly distinguishable from the background. In this case, it is easy to choose a threshold value by taking the value that is in the valley between two peaks of the histogram. However, in the real world, the gray level histograms of the images are always multi-modal and; hence, it is not simple to determine the exact locations of distinct valleys in multi-modal histograms. Thanks to growing of evolutionary algorithm, the problem of finding the optimal threshold makes easier considering a specific target function [27]. Vantaram and Saber [23] have presented a thorough survey of a variety of thresholding techniques, among which global histogram based algorithms are widely employed to determine the threshold. This global thresholding technique can be classified into parametric and nonparametric approaches. In parametric approaches [5], the gray level distribution of each class has a probability density function that follows a Gaussian distribution. This method is computationally expensive and time-consuming. Nonparametric approaches determine the threshold values in an optimal fashion based on a given criterion. The nonparametric approaches such as Otsu [17] and Kapur [11] are robust and more accurate than the parametric methods.

The Otsu and Kapur methods can be easily extended to multilevel thresholding problem but inefficient in determining the optimal thresholds due to the exponential growth in computation time. To improve the efficiency, many methods have been proposed for solving the multilevel thresholding problem [35]. Liao et al. [13] showed that the recursive algorithm significantly reduces the computational complexity of determining the multi-level thresholds by accessing a look-up table when compared with conventional Otsu and Kapur methods. However, it still suffers from the problem of a significant processing time when the number of thresholds increases.

To eliminate such problems, numerous works on the topic has been presented based on swarm algorithms, including Genetic Algorithm (GA) [7], Particle Swarm Optimization (PSO), Artificial Bee Colony (ABC) [1], Ant Colony Optimization (ACO) [21], Bacterial Foraging (BF) [19], and Honey Bee Mating Optimization (HBMO) [9]. Yin et al. [29] have proposed a PSO based multilevel minimum cross entropy threshold selection method to achieve near-optimal thresholds. Zhang and Wu [33] used ABC algorithm for optimizing Tsallis entropy. After that, Akay [17] employed PSO and ABC to find the optimal multilevel thresholds. Kapur's entropy, one of the maximum entropy techniques, and between-class variance has been investigated as fitness functions. The results of performing this algorithm on a set of test image, using various numbers of thresholds, were assessed using statistical tools and suggest that Otsu's technique, PSO and ABC show equal performance when the number of thresholds is two, while the ABC algorithm performs better than PSO and Otsu's technique when the number of thresholds is greater than two.

Yang and Deb [26] formulated a new meta-heuristic algorithm, called cuckoo search algorithm which is based on the interesting breeding behavior such as brood parasitism of particular species of Cuckoos, and the preliminary studies show that it is very promising and could outperform the existing algorithms such as GA, PSO, ABC, ACO, BF and HBMO [5].

All of these methods have their limitations concerning convergence speed or accuracy, and all researchers aim at finding a tradeoff between these two important aspects. Besides the mentioned limitations, there is a shortcoming with which escaping from is rather impossible; it is local optima. The main idea behind this research is to cope with this deficiency while increasing the accuracy of optimization. It is evident that each optimization may face different local solutions; hence, merging two optimization algorithms has a high capability of escaping from individual local solutions. To reach a robust multilevel thresholding, BF, with its accuracy and Harmony Search (HS), with its convergence speed, [3] are joined to be able of approaching the problem of mentioned local optimum. To do switching between BF and HS, we have investigated the power of a sort of reinforcement learning algorithm, namely Learning Automata (LA) [16], to give a reward in case of right behavior as well as providing a penalty when the algorithm faces local optimum. The presented scheme is used for maximizing the Kapur's entropy and Otsu. Experimental image thresholding results are obtained for qualitative analysis. Quantitative results are defined by peak signal-to-noise ratio (PSNR), structural similarity index (SSIM) and feature similarity index (FSIM). The performance improvement of the proposed algorithm, so-called BFHS, based segmentation approach is measured regarding PSNR, SSIM, and FSIM, in comparison with the state of the art techniques.

The rest of this paper is organized as follows. Section 2 dedicates to formulate the foundations of multilevel thresholding. Brief explanations of the algorithms that are used in this study are given in Section 3. Implementation of the hybrid method is described in Section 4. The experimental results are provided in Section 5, and, finally, the conclusion remarks are given in Section 6.

## 2 Problem formulation for multilevel thresholding

The optimal thresholding methods search the thresholds such that the segmented classes on the histogram satisfy the desired property. This is performed by maximizing an objective function which uses the selected thresholds as the parameters. In this paper, two broadly used optimal thresholding methods namely entropy criterion (Kapur) method and between-class variance (Otsu) methods are used.

### 2.1 Kapur method

The entropy criterion method has been employed in determining whether the optimal thresholding can provide histogram-based image segmentation with satisfactory desired characteristics [11, 17]. Kapur has developed the algorithm for bi-level thresholding, and this bi-level thresholding can be described as follows:

Let there be $L$ gray levels in a given image and let them be in a range $\{0,1,2, \ldots,(L-1)\}$. Then one can define $p_{i}=h(i) \times N,(0 \leq i \leq(L-1))$ where $h(i)$ denotes the number of pixels for the corresponding gray-level $L$, and $N$ denotes total number of pixels in the image which is equal to $\sum_{i=0}^{L-1} h(i)$. Then the objective is to maximize the fitness function.

$$
\begin{equation*}
f(t)=H_{0}+H_{1} \tag{1}
\end{equation*}
$$

where $H_{0}=-\sum_{i=0}^{t-1}\left(\frac{p_{i}}{\omega_{0}} \ln \frac{p_{i}}{\omega_{0}}\right), \omega_{0}=\sum_{i=0}^{t-1} P_{i}, H_{1}=-\sum_{i=t}^{L-1}\left(\frac{p_{i}}{\omega_{1}} \ln \frac{p_{i}}{\omega_{1}}\right), \omega_{1}=\sum_{i=t}^{L-1} p_{i}$. The optimal threshold is the gray level that maximizes Eq. 1. This Kapur's entropy criterion method tries to achieve a centralized distribution for each histogram-based segmented region of the image. This method is extended to multilevel thresholding as follows:

The optimal multilevel thresholding problem can be configured as an $m$-dimensional optimization problem, for determination of $m$ optimal thresholds for a given image $\left[t_{1}, t_{2}\right.$, $\left.\ldots, t_{\mathrm{m}}\right]$, where the aim is to maximize the objective function:

$$
\begin{equation*}
f\left(\left[t_{1}, t_{2}, \ldots, t_{m}\right]\right)=H_{0}+H_{1}+\ldots+H_{m} \tag{2}
\end{equation*}
$$

where $H_{0}=-\sum_{i=0}^{t_{i}-1}\left(\frac{p_{i}}{\omega_{0}} \ln \frac{p_{i}}{\omega_{0}}\right), \omega_{0}=\sum_{i=0}^{t_{1}-1} p_{i}, \ldots, H_{m}=-\sum_{i=t_{m}}^{L-1}\left(\frac{p_{i}}{\omega_{m}} \ln \frac{p_{i}}{\omega_{m}}\right), \omega_{m}=\sum_{i=t_{m}}^{L-1} p_{i}$.

### 2.2 Otsu method

Thresholding using Otsu's method is a nonparametric segmentation technique, which is used to segment the entire image into many regions; as a result, the variance of the various classes can be maximized. Between-class variance was proposed by [17] as sum of sigma functions of each class and is defined by Eq. (3):

$$
\begin{gather*}
f(t)=\sigma_{0}+\sigma_{1}  \tag{3}\\
\sigma_{0}=\omega_{0}\left(\mu_{0}-\mu_{T}\right)^{2} \cdot \sigma_{1}=\omega_{1}\left(\mu_{1}-\mu_{T}\right)^{2} \tag{4}
\end{gather*}
$$

where $\mu_{T}$ represents the mean intensity of input image. In the case of bi-level thresholding, the average level of each class $\left(\mu_{i}\right)$, can be obtained using Eq. (5):

$$
\begin{equation*}
\mu_{0}=\sum_{i=0}^{t-1} \frac{i p_{i}}{\omega_{0}} . \mu_{1}=\sum_{i=t}^{L-1} \frac{i p_{i}}{\omega_{1}} . \tag{5}
\end{equation*}
$$

By maximizing the between-class ${ }_{*}$ variance function, the optimal threshold value can be achieved using Eq. (6): $\quad t^{*}=\operatorname{argmax}(f(t))$

Furthermore, between-class-variance [1] is extended to multilevel thresholding problem as followed by Eq. (7):

$$
\begin{equation*}
f(t)=\sum_{i=0}^{m} \sigma_{i} \tag{7}
\end{equation*}
$$

The sigma term and the mean levels can be obtained from Eq. (8):

$$
\begin{align*}
& \sigma_{0}=\omega_{0}\left(\mu_{0}-\mu_{T}\right)^{2} \cdot \sigma_{1}=\omega_{1}\left(\mu_{1}-\mu_{T}\right)^{2} \cdot \sigma_{j}=\omega_{j}\left(\mu_{j}-\mu_{T}\right)^{2} \cdot \sigma_{m}=\omega_{m}\left(\mu_{m}-\mu_{T}\right)^{2} \\
& \mu_{0}=\sum_{i=0}^{t_{1}-1} \frac{i p_{i}}{\omega_{0}} \cdot \mu_{1}=\sum_{i=t_{1}}^{t_{2}-1} \frac{i p_{i}}{\omega_{1}} \cdot \mu_{j}=\sum_{i=t_{j}}^{t_{j+1}-1} \frac{i p_{i}}{\omega_{j}} \cdot \mu_{m}=\sum_{i=t_{m}}^{L-1} \frac{p_{i}}{\omega_{m}} . \tag{8}
\end{align*}
$$

The optimal multilevel thresholding is configured by maximizing the objective function using Eq. (9):

$$
\begin{equation*}
(\vec{t})^{*}=\operatorname{argmax}\left(\sum_{i=0}^{m} \sigma_{i}\right) \tag{9}
\end{equation*}
$$

The Kapur and Otsu methods have been proven as an efficient method for bi-level thresholding in image segmentation. However, when these methods are extended to multilevel thresholding, the computation time grows exponentially with the number of thresholds. It would limit the multilevel thresholding applications. To overcome the above problem, this paper proposes a hybrid Bio-Inspired learning algorithm for solving multi-level thresholding problem. The aim of the proposed method is to maximize the Kapur and Otsu objective functions.

## 3 Brief explanations of the algorithms in the study

Optimization is the process of making something better than the previous form. Over the last decade, the aggregate intelligent behavior of insect or animal groups in the natural world for example flocks of birds, colonies of ants, schools of fish, swarms of bees, and termites have fascinated the interest of researchers. The collective action of insects, birds or animals is identified as swarm behavior. Many researchers have used swarm behavior as a framework for solving complicated real-world problems.

The aim of this research is to apply the accuracy of BF and the speed of HS to satisfy the target function. LA acts as the intelligent part of the algorithm and, on current conditions of the problem, selects one of the two algorithms so as to reach a satisfaction in optimizing target function.

### 3.1 Bacterial foraging optimization algorithm

The Bacterial Foraging Optimization Algorithm (BF) [3] is inspired by the group foraging behavior of bacteria such as E.coli and M.xanthus. Specifically, the BF is inspired by the chemotaxis behavior of bacteria that will perceive chemical gradients in the environment (such as nutrients) and move toward or away from specific signals.

The information processing strategy of the algorithm is to allow cells to stochastically and collectively swarm toward the ideal situations. This is achieved through a series of three processes on a population of simulated cells: 1) 'Chemotaxis' where the cost of cells is derated by the proximity to other cells and cells move along the manipulated cost surface one at a time (the majority of the work of the algorithm), 2) 'Reproduction' where only those cells that performed well over their lifetime may contribute to the next generation, and 3) 'Elimination-dispersal' where cells are discarded, and new random samples are inserted with a low probability.

A bacteria cost is derated by its interaction with other cells and is calculated as follows:

$$
\begin{align*}
& g\left(\text { cell }_{k}\right)=\sum_{i=1}^{S}\left[-d_{\text {attr }} \times \exp \left(-w_{\text {attr }} \times \sum_{m=1}^{P}\left(\text { cell }_{m}^{k}-\text { other }_{m}^{i}\right)^{2}\right)\right]  \tag{10}\\
& +\sum_{i=1}^{S}\left[-h_{\text {repel }} \times \exp \left(-w_{\text {repel }} \times \sum_{m=1}^{P}\left(\text { cell }_{m}^{k}-\text { other }_{m}^{i}\right)^{2}\right)\right]
\end{align*}
$$

where $c e l l_{k}$ is a given cell, $d_{\text {attr }}$ and $w_{\text {attr }}$ are attraction coefficients, $h_{\text {repel }}$ and $w_{\text {repel }}$ are repulsion coefficients, $S$ is the number of cells in the population, $P$ is the number of dimensions on a given cells position vector.

The remaining parameters of the algorithm are as follows: Cells num is the number of cells maintained in the population, $N_{e d}$ is the number of elimination-dispersal steps, $N_{r e}$ is the number of reproduction steps, $N_{c}$ is the number of chemotaxis steps, $N_{s}$ is the number of swim steps for a given cell, Step $_{\text {size }}$ is a random direction vector with the same number of dimensions as the problem space, and each value $\in[-1,1]$, and $P_{e d}$ is the probability of a cell being subjected to elimination and dispersal.

### 3.2 Harmony search algorithm

Harmony Search [3] was inspired by the improvisation of Jazz musicians. Specifically, the process in which the musicians (who may have never played together before) rapidly refine their individual improvisation through variation resulting in an aesthetic harmony.

The information processing objective is achieved by stochastically creating candidate solutions in a step-wise fashion, where each element is either pulled out randomly from a memory of high-tone results, adjusted from the retention of high-quality solutions or assigned randomly within the bounds of the problem. The memory of candidate solutions is initially random, and a greedy acceptance criterion is employed to admit new candidate solutions only if they sustain an improved objective value, substituting an existing member.

### 3.3 Learning automata

LA operates by selecting actions via a stochastic process. Such actions operate within an environment while being assessed according to a measure of the system performance. Figure 1a shows the typical learning system architecture. The automaton selects an action ( $\mathbf{X}$ ) probabilistically. Such actions are applied to the environment, and the performance evaluation function provides a reinforcement signal $\beta$. This is used to update the automaton's internal probability distribution whereby actions that achieve desirable performance are reinforced via an increased probability. Likewise, those underperforming actions are penalized or left unchanged depending on the particular learning rule which has been employed. Over time, the average performance of the system will improve until a given limit is reached. Regarding optimization problems, the action with the highest probability would correspond to the global minimum as demonstrated by rigorous proofs of convergence available in [10, 16].

A wide variety of learning rules has been reported in the literature. One of the most widely used algorithms is the linear reward/penalty $\left(L_{R P}\right)$ scheme, which has been shown to guarantee convergence properties (see [16]). With a large number of discrete actions, the probability of selecting any particular action becomes low and the convergence time can become excessive. To avoid this, LA can be connected in a parallel setup like the one shown in Fig. 2b. Each automaton operates a smaller number of actions, and the 'team' works together in a cooperative manner. This scheme can also be used where multiple actions are required.

If action $x$ (parameter) is defined over the range $\left(x_{\min }, x_{\max }\right)$, the probability density function $f(x, n)$ at iteration $n$ is updated according to the following rule:

$$
f(x . n+1)=\left\{\begin{array}{cl}
\alpha[f(x . n)+\beta(n) H(x . r)] & \text { if } x \in\left(x_{\min } \cdot x_{\max }\right)  \tag{11}\\
0 & \text { otherwise }
\end{array}\right.
$$

where $\alpha$ is chosen to re-normalize the distribution according to the following condition:

$$
\begin{equation*}
\int_{x_{\min }}^{\max } f(x . n+1) d x=1 \tag{12}
\end{equation*}
$$



Fig. 2 (a) Reinforcement learning system and (b) Interconnected automata
where $\beta(n)$ is the reinforcement signal from the performance evaluation and $H(x, r)$ is a symmetric Gaussian neighborhood function centered on $r=x(\mathrm{n})$. It yields

$$
\begin{equation*}
H(x . r)=\lambda \times \exp \left(-\frac{(x-r)^{2}}{2 \sigma^{2}}\right) \tag{13}
\end{equation*}
$$

where $\lambda$ and $\sigma$ are parameters that determine the height and width of the neighborhood function. They are defined in terms of the range of actions as follows:

$$
\begin{equation*}
\sigma=g_{w} \times\left(x_{\max }-x_{\min }\right), \quad \lambda=\frac{g_{h}}{\left(x_{\max }-x_{\min }\right)} \tag{14}
\end{equation*}
$$

Free parameters thus control the speed and resolution of learning $g_{w}$ and $g_{h}$. Let action $x(n)$ be applied to the environment at iteration $n$, returning a cost or performance index $J(n)$. Current and previous costs are stored as a reference set $R(n)$. The median and minimum values $J_{\text {med }}$ and $J_{\text {min }}$ may thus be calculated by means of $\beta(n)$, which is defined as follows:

$$
\begin{equation*}
\beta(n)=\max \left\{0 \cdot \frac{J_{\text {med }}-J(n)}{J_{\text {med }}-J_{\text {min }}}\right\} \tag{15}
\end{equation*}
$$

To avoid problems with infinite storage requirements and to allow the system to adapt to changing environments, only the last $m$ values of the cost functions are stored in $R(n)$. Eq. (15) limits $\beta(n)$ to values between 0 and 1 and only returns nonzero values for those costs that are below the median value. It is easy to understand how $\beta(n)$ affects the learning process as follows: during the learning, the performance and the number of selecting actions can be wildly variable, generating extremely high computing costs. However, $\beta(n)$ is insensitive to such extremes and high values of $J(n)$ resulting from a poor choice of actions. As the learning continues, the automaton converges towards more worthy regions of the parameter space as such actions are chosen to be evaluated more often. When more of such responses are being received, $J_{\text {med }}$ gets reduced. Decreasing $J_{\text {med }}$ in $\beta(n)$ effectively enables the automaton to refine its reference around better responses (previously received), and hence resulting in a better discrimination between selected actions.

In order to define an action value $x(n)$ which has been associated with a given probability density function, a uniformly distributed pseudo-random number $z(n)$ is generated within the range of $[0,1]$. Simple interpolation is thus employed to equate this value to the cumulative distribution function:

$$
\begin{equation*}
\int_{x_{\min }}^{x(n)} f(x . n) d x=z(n) \tag{16}
\end{equation*}
$$

## 4 Implementation of the Hybrid Method

In the proposed method a pixel is randomly selected to assign it to each member of the hybrid method, from now on BFHS, the population as a position of the pixel. In this study, four different pixel classes are used to segment the images and the idea is to show the effectiveness of the algorithm and its performance against other algorithms solving the same task. However, the implementation can easily be transferred to cases with a greater number of pixel classes.

To approach the histogram of an image by Kapur and Otsu functions, it is necessary to calculate the optimum values of the parameters for each function. This problem can be solved by optimizing Eq. (2) and Eq. (9), the initial values of the relevant parameters are summarized in Table 1.

In the LA optimization, each parameter is considered like an Automaton, which can choose actions. Such actions correspond to values assigned to the parameters by a probability distribution within the interval. For this 2-dimensional problem, one automaton will be created to represent the parametric approach of the corresponding histogram. One of the main advantages of the LA algorithm regarding multi-dimensional problems is that the automatons are coupled only through the environment, which is considered of type $P$ in this study.

As a matter of fact, if each of automata's actions fails, it will be punished, and if each of them is successful, it will get a reward. An action fails when it cannot improve $A(n)=\left\{s, N_{c}\right.$, $\left.N_{s}, N_{\text {re }}, N_{\text {ed }}, d_{\text {attract }}, w_{\text {attract }}, h_{\text {repellent }}, w_{\text {repellent }}, P_{\text {ed }}, \mathrm{HM}, \mathrm{HMCR}, \mathrm{PAR}, \mathrm{BW}, \mathrm{NI}\right\}$ in an iteration, therefore, it will be punished. If an action can improve $A(n)$, it will get a reward. At the beginning of performing the proposed algorithm, LA probability vector is equal for both actions. BF action is chosen with $50 \%$ probability $[0 \ldots 50]$ and also HS action is selected with $50 \%$ probability ( $50 \ldots$ 100]. In fact, given probabilities are both set to $50 \%$. LA generates a random number with uniform distribution. Regarding the random number and probability vector of LA, one of the actions is selected. If the chosen action is successful, it gets the reward, and its probability value increases according to learning algorithm. However, if it fails, its probability will be decreased accordingly. Therefore, LA learns in different conditions to find out which action is better to be executed more.

The quality of the approach is converted into a reinforcement signal $\beta(n)$ (through Eq. 15). After the reinforcement value $\beta(n)$ is defined as a product of the elected approach $A(n)$, the distribution of probability is updated for $n+1$ of each automaton (according to the Eq. 11). To simplify parameters in Eq. (14), they will take the same

Table 1 Initial parameters of BFHS

| Method | Parameter | Value |
| :--- | :--- | :--- |
| Bacterial Foraging (BF) | Number of bacterium $(s)$ | 20 |
|  | Number of chemotactic steps $\left(N_{c}\right)$ | 10 |
|  | Swimming length $\left(N_{s}\right)$ | 10 |
|  | Number of reproduction steps $\left(N_{r e}\right)$ | 4 |
|  | Number of elimination of dispersal events $\left(N_{e d}\right)$ | 2 |
|  | Depth of attract $\left(d_{\text {attract }}\right)$ | 0.1 |
|  | Width of attract $\left(w_{\text {attract }}\right)$ | 0.2 |
|  | Height of repellent $\left(h_{\text {repellent }}\right)$ | 0.1 |
|  | Width of repellent $\left(w_{\text {repellent }}\right)$ | 10 |
| Harmony Search (HS) | Probability of elimination and dispersal $\left(P_{e d}\right)$ | 0.02 |
|  | Harmony memory (HM) | 100 |
|  | Harmony memory consideration rate $(\mathrm{HMCR})$ | 0.75 |
|  | Pitch adjusting rate (PAR) | 0.5 |
|  | Distance bandwidth (BW) | 0.3 |
|  | Number of improvisations (NI) | 250 |

value for the automaton, such that $g_{w}=0.02$ and $g_{h}=0.3$. In this work, the optimization process considers a limit up to 2000 iterations.

The final step is to determine the optimal threshold values $T_{i}$, just as it is illustrated in Fig. 3. The optimization algorithm can thus be described as follows (see Table 2):

## 5 Results and discussions

In this research, two sets of experiments were conducted. In the first set, experiments are done on ten benchmark gray-scale images, Lenna, Pepper, Baboon, Hunter, Map, Cameraman, Living room, House, Airplane, Butterfly (refer to Fig. 4), with size of $512 \times 512$ and the uniformity metric (Eq. 17) [12] is used in order to compare image segmentation performance. Whereas in the second set, experiments are done on a set of satellite images [31], The obtained results are then compared with HS, BF and GA as shown visually in Figs. 7, 8, 9, 10, 11 and 12 and quantitatively in Tables 4, 5, 6, 7, 8 and 9.

### 5.1 First set of experiments

The hybrid algorithm (BFHS) along with BF, KPSO [22], and GA are used to segment these benchmark images. BF and KPSO parameters are adjusted according to [22]. GA parameters are adjusted on [28]. The region uniformity is first stated by Levine and Nazif [12], which is primarily used as a criterion for measuring the quality of image segmentation. The uniformity of a feature over a region is defined as being inversely proportional to the variance of the values of that feature evaluated, at every pixel belonging to that region, with an appropriate weighting factor.

$$
\begin{equation*}
u=1-2 \times c \times \frac{\sum_{j=0}^{c} \sum_{i \in R_{j}}\left(f_{i}-\mu_{j}\right)^{2}}{N \times\left(f_{\max }-f_{\min }\right)^{2}} \tag{17}
\end{equation*}
$$

where $c$ is the number of thresholds. $R_{j}$ is the segmented region $j . N$ is the total number of pixels in the given image. $f_{\mathrm{i}}$ shows the gray level of pixel $i . \mu_{\mathrm{i}}$ is the mean gray level of pixels

Fig. 3 Thresholding points determination


Table 2 Pseudo-code of the hybrid method
1: Initialization
Initialize $A(n)=\left\{s, N_{c}, N_{s}, N_{r e}, N_{\text {ed }}, d_{\text {attract }}, w_{\text {attract }}, h_{\text {repellent }}, w_{\text {repellent }}, P_{\text {ed }}\right.$, HM, HMCR, PAR,
BW, NI $\}$ to the parameters of Table 1, set iteration number $n=0, g_{w}=0.02, g_{h}=0.3$
$A=\arg \min J(n)$
Repeat
$L A$ select an action $i$ based on the probability vector $p$
If selected action is $B F$ procedure, then
Loop $l=l+1 / /$ Elimination-dispersal loop
Loop $k=k+1 / /$ Reproduction loop
Loop $\mathrm{j}=j+1 / /$ Chemotaxis loop
For $\mathrm{i}=1,2, \ldots, s$ takes a chemotactic step for bacterium $i$ as follows:
Compute the value fitness function using Eq.
(10). Let $P_{\text {best }}=g(i, j, k, l)$ to save this value since we may find a better cost via the run.
Tumble: Generate a random vector $\Delta(i)$ with each element $\Delta_{m}(i), m=1,2, \ldots, P$, a random number on $[-1,1]$.
do MOVE
do SWIM
$m=0$
While $m<N_{s}$

$$
m=m+1 .
$$

$$
\text { If } g(i, j+1, k, l)<P_{\text {best }} \text {, then }
$$

$P_{\text {best }}=g(i, j+1, k, l)$
other $^{i}(j+1 . k . l)$
$=$ other $^{i}(j+1 . k . l)$
$+C(i) \frac{\Delta(i)}{\sqrt{\Delta^{T}(i) \Delta(i)}}$
// where $C(i)$ is the direction of the tumble for bacterium $i$
Else, $m=N_{s}$.
go to $\mathbf{1 0}$ to process the next bacterium.

## End Loop

Step 5. Perform reproduction and elimination-dispersal operation.

## End Loop

## End Loop

If the maximum number of chemotactic, reproduction and elimination dispersal steps are reached, then go to $\mathbf{2 6}$. Otherwise, go to $\mathbf{6}$.
$A(n)=P_{\text {best }}$
elseif selected action is $H S$ then
Improvise a new harmony $\mathbf{x}_{\text {new }}$ as follows:
While $j^{<n}$ do
If $r_{1}<$ HCMR then
$\mathrm{x}_{\text {new }}(j)=\mathrm{x}_{a}(j)$ where $a \in(1,2, \ldots$, HMS $)$
If $r_{2}<$ PAR then
$\mathrm{x}_{\text {new }}(j)=\mathrm{x}_{a}(j) \pm r_{3} \times \mathrm{BW}$
$/ / r_{l}, r_{2}, r_{3} \in \operatorname{rand}(0,1)$
If $\mathrm{x}_{\text {new }}(j)<l(j)$ then
(Continued)

| 35: | $\mathrm{x}_{\text {new }}(j)=l(j)$ |
| :---: | :---: |
|  | // $l$ is the lower bound |
| 36: | If $\mathrm{x}_{\text {new }}(j)>u(j)$ |
| 37: | $\mathrm{x}_{\text {new }}(j)=u(j)$ |
|  | // $u$ is the upper bound |
| 38: | else |
| 39. | $\mathrm{x}_{\text {new }}(j)=l(j) \pm r \times(u(j)-l(j))$ |
| 39: | $/ / r \in \operatorname{rand}(0,1)$ |
| 40: | End Loop |
| 41: | Update the HS as $\mathbf{x}_{\text {worst }}=\mathbf{x}_{\text {new }}$ if $f\left(\mathbf{x}_{\text {new }}\right)>f\left(\mathbf{x}_{\text {worsst }}\right)$ |
| 42: | If $N I$ is completed or the stop criterion is met, jump to 43; else go to $\mathbf{2 8}$. |
| 43: | $A(n)=\mathbf{x}_{\text {best }}$ |
| 44: | Update probability vector $p$ of $L A$ using learning rule. |
| 45: | Obtain the minimum, $J_{\text {min }}$, and median, $J_{\text {med }}$ of $J(n)$. |
| 46: | Evaluate $\beta(n)$ via Eq. (15). |
| 47: | <2000 |

in the $j$ th region. $f_{\min }$ and $f_{\max }$ are the minimum and maximum gray level of pixels in the given image, respectively. Typically, $u \in[0,1]$ and a larger amount for $u$ declares that the thresholds are specified with better quality on the histogram.

Figure 5 shows segmented images using the proposed algorithm with 5, 4, and 3 thresholds, respectively. Average uniformity obtained from algorithms on these benchmark images with thresholds of $2,3,4$, and 5 are tabulated in Table 3.

Due to the low ability in local search, GA algorithm leads to the worst results for all cases. Moreover, obtained results from KPSO are not better than BFHS for all cases. The reason is that occasionally KPSO converges toward a local optimum and thus, obtained results are not appropriate. Although BFHS is responsible for exiting from local optimum, it sometimes may not be successful.

Indeed, the inappropriate result of KPSO causes fast convergence of particles to a local optimum. Obtained results from the proposed algorithm are better than other algorithms in all cases. Basically, in the proposed algorithm, LA tries to perform an approach which involves the best result according to the current conditions of the algorithm in the optimization process. Hence, when the problem conditions are such that an algorithm would not be able to improve optimization process, it is used less by LA. As a result, both BF and HS algorithms abilities are utilized in the proposed algorithm. It is observed in the course of experiments that the uniformity amount is improved by increasing the number of thresholds for all algorithms.

### 5.2 Second set of experiments

In this part of experiments, a distinct study on the application of BFHS, BF, HS and GA with two different objective functions (Kapur and Otsu) is made for multilevel thresholding for image segmentation. For the entire test of satellite images that have been considered (see Fig. 6), the BFHS performs as well as or is better than the BF,


Fig. 4 Standard images which are used for testing the hybrid method. (a) Lenna, (b) Pepper, (c) Baboon, (d) Hunter, (e) Map, (f) Cameraman, (g) Living room, (h) House, (i) Airplane, (j) Butterfly

HS, and GA. The experimental results provide evidence for outstanding performance, accuracy and convergence of the proposed algorithm in comparison to other methods.


Fig. 5 Thresholded images obtained by the hybrid method on Kapur thresholding (a)-(j) represents 3-level thresholding, ( $a^{\prime}$ )-(j') represents 4-level thresholding, (a")-(j") represents 5-level thresholding

On the other hand, it is proved that the computational cost of BFHS is lower than other evolutionary approaches used in the comparison Fig. 7.


Fig. 5 (continued)


Fig. 5 (continued)

Although, satellite images often need segmentation in the presence of uncertainty, caused due to the factors like highly dependent on environmental conditions, poor resolution, and poor illumination, and have a very low spatial resolution. Owing to the presence of different bands with different wavelength region in the satellite images, the efficiency of the algorithms is affected and containing high resolution is one more cause of inefficiency. As a result, it leads to computational complexity during segmentation.

In satellite images, the rate of information is very high because of that existing features in the image is very dense. Due to that, the rate of change from one region to another region is very rapid. Therefore, in the case of segmentation of remote sensing images or satellite images, accurate segmentation is a very challenging task.

To achieve an accurate and fast segmentation of satellite images, BFHS based robust technique with two most popular objective functions of multilevel thresholding are utilized in this paper, which shows the effectiveness of their segmented results.

While estimating the segmented images, PSNR gives the similarity of an image against a reference image based on the mean square error (MSE) of each pixel which was also reported in [2]. The SSIM is used to compare the structure of original and thresholded image [24]. The SSIM index is calculated from [2]. FSIM [34] is used to calculate the similarity between two images, which can be calculated from [2].

### 5.2.1 Based on Kapur's entropy

In this section, the results acquired for various satellite images using Kapur's objective function are discussed. Table 4 depicts the number of thresholds, objective values and corresponding optimal threshold values obtained by BFHS, BF, HS and GA methods. It is examined that the objective value evaluated by the proposed method yields highest among all the techniques being compared to different satellite images. PSNR (dB) and MSE values obtained using the proposed BFHS based method are listed in Table 5 and compared with the result acquired using BF, HS, and GA methods respectively. Despite the quality estimation factor using PSNR and MSE, algorithm efficiency (CPU Timing (in seconds)) and feature measurement assessment parameters are also checked using FSIM and SSIM, which is shown in Table 6. It can be clearly observed from Tables 4, 5 and 6 that the proposed BFHS-Kapur's

Table 3 Comparison of uniformity of the proposed algorithm and four comparative approaches

| Image | T | GA | BF | KPSO | BFHS |
| :---: | :---: | :---: | :---: | :---: | :---: |
| Lenna | 2 | 0.8844 | 0.9730 | 0.9728 | 0.9781 |
|  | 3 | 0.9164 | 0.9781 | 0.9783 | 0.9802 |
|  | 4 | 0.9198 | 0.9816 | 0.9811 | 0.9833 |
|  | 5 | 0.9269 | 0.9835 | 0.9834 | 0.9841 |
| Pepper | 2 | 0.8659 | 0.9719 | 0.9720 | 0.9729 |
|  | 3 | 0.8970 | 0.9773 | 0.9771 | 0.9785 |
|  | 4 | 0.9054 | 0.9784 | 0.9769 | 0.9793 |
|  | 5 | 0.9080 | 0.9814 | 0.9825 | 0.9825 |
| Baboon | 2 | 0.8534 | 0.9720 | 0.9731 | 0.9792 |
|  | 3 | 0.8705 | 0.9759 | 0.9752 | 0.9803 |
|  | 4 | 0.8812 | 0.9801 | 0.9772 | 0.9829 |
|  | 5 | 0.8944 | 0.9831 | 0.9838 | 0.9862 |
| Hunter | 2 | 0.8545 | 0.9722 | 0.9732 | 0.9793 |
|  | 3 | 0.8718 | 0.9765 | 0.9762 | 0.9812 |
|  | 4 | 0.8812 | 0.9804 | 0.9777 | 0.9832 |
|  | 5 | 0.8961 | 0.9839 | 0.9838 | 0.9862 |
| Map | 2 | 0.8546 | 0.9729 | 0.9732 | 0.9795 |
|  | 3 | 0.8732 | 0.9770 | 0.9764 | 0.9815 |
|  | 4 | 0.8837 | 0.9806 | 0.9786 | 0.9839 |
|  | 5 | 0.8996 | 0.9841 | 0.9838 | 0.9868 |
| Cameraman | 2 | 0.8591 | 0.9736 | 0.9736 | 0.9799 |
|  | 3 | 0.8751 | 0.9783 | 0.9767 | 0.9816 |
|  | 4 | 0.8850 | 0.9811 | 0.9794 | 0.9840 |
|  | 5 | 0.8996 | 0.9842 | 0.9840 | 0.9878 |
| Living room | 2 | 0.8592 | 0.9736 | 0.9746 | 0.9800 |
|  | 3 | 0.8801 | 0.9785 | 0.9767 | 0.9817 |
|  | 4 | 0.8868 | 0.9816 | 0.9798 | 0.9842 |
|  | 5 | 89.96 | 0.9844 | 0.9851 | 0.9880 |
| House | 2 | 0.8638 | 0.9738 | 0.9747 | 0.9801 |
|  | 3 | 0.8803 | 0.9796 | 0.9768 | 0.9820 |
|  | 4 | 0.8896 | 0.9826 | 0.9821 | 0.9842 |
|  | 5 | 0.9020 | 0.9845 | 0.9855 | 0.9880 |
| Airplane | 2 | 0.8641 | 0.9753 | 0.9750 | 0.9802 |
|  | 3 | 0.8810 | 0.9800 | 0.9771 | 0.9828 |
|  | 4 | 0.8924 | 0.9827 | 0.9826 | 0.9846 |
|  | 5 | 0.9073 | 0.9848 | 0.9866 | 0.9882 |
| Butterfly | 2 | 0.8641 | 0.9753 | 0.9750 | 0.9802 |
|  | 3 | 0.8808 | 0.9800 | 0.9772 | 0.9829 |
|  | 4 | 0.9031 | 0.9828 | 0.9873 | 0.9850 |
|  | 5 | 0.9124 | 0.9849 | 0.9874 | 0.9885 |

based technique offers superior objective values in comparison with other evolutionary algorithms such as BF, HS, and GA.


Fig. 6 Five different satellite images are used in the experiments. (a-e) represent original satellite images, (f-j) illustrate corresponding histogram image

### 5.2.2 Based on between-class variance

The performance evaluation of Otsu approach for numerous satellite images is discussed. The number of thresholds, objective values, and corresponding optimal thresholds determined using proposed BFHS technique are listed in Table 7, and results are compared with those obtained using BF, HS, and GA methods respectively. PSNR (dB) and MSE values obtained using the proposed BFHS based method are listed in Table 8 and compared with the results acquired using BF, HS and GA methods, respectively. High PSNR values might be obtained from methods that minimize MSE. Indeed, PSNR gives the similarity of an image against a reference image based on the MSE of each pixel. Apart from quality measurement using PSNR and MSE, algorithm efficiency (CPU time) and feature measurement assessment (FSIM and SSIM) are also checked and presented in Table 9. It can be evidently realized from Tables 7, 8 and 9 that the proposed BFHS-Otsu's based technique provides higher objective values than the other evolutionary algorithms like BF, HS, and GA. Figure 8 shows the segmented images for different threshold levels ( $m=2-5$ ) obtained for BFHS-Otsu, BF-Otsu, HS-Otsu, and GA-Otsu. Figures 9, 10, 11 and 12 show the segmented images for various threshold levels ( $m=2-5$ ) obtained for BFHSOtsu.

### 5.3 Convergence and computational cost

The convergence curves of BF-Otsu, HS-Otsu, and BFHS-Otsu algorithms have been plotted for the Lenna image for various threshold levels ( $m=2-5$ ), as shown in Fig. 13. Convergence curves are fairly identical in their plots of HS, BF, and BFHS algorithms. These show that for different thresholds values, BF algorithm converges at the slowest speed and locates local optima only. The HS has fastest convergence speed but fails to locate the global optima. The BFHS not only has fast convergence but also managed to locate the true global optima.

Results of $\mathbf{1}^{\text {st }}$ Test Image using Kapur's Entropy


Fig. 7 Results of 1 st test satellite image using BFHS, BF, HS and genetic algorithm (GA) with Kapur's entropy. (a-d) 2-level to 5 -level thresholding based segmented image with the best thresholds obtained from BFHS algorithm using Kapur's entropy criterion, (e-h) 2-level to 5-level thresholding based segmented image with the best thresholds obtained from BF algorithm using Kapur's entropy criterion, (i-l) 2-level to 5-level thresholding based segmented image with the best thresholds obtained from HS algorithm using Kapur's entropy criterion and $(\mathbf{m}-\mathbf{p})$ 2-level to 5-level thresholding based segmented image with the best thresholds obtained from GA using Kapur's entropy criterion

Due to the fact that several learning algorithms are sensitive and dependent on the initial value of parameters, it is worth taking this dependency into account. In this experiment, initial values for all methods are initialized in different values while the same histogram is considered for the approximation task. Two sets of randomized values using a uniform distribution were generated to set the initial values of the BF, HS, and BFHS.
Table 4 Comparison of best objective function values and their corresponding threshold values between BFHS, BF, HS, and GA-based technique using Kapur's entropy

| Test images | $m$ | Best objective function values |  |  |  | Optimum threshold values |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  | BFHS | BF | HS | GA | BFHS | BF | HS | GA |
| $1(750 \times 750)$ | 2 | 0.898988 | 0.898986 | 0.898972 | 0.898912 | 62, 180 | 65, 183 | 115, 136 | 31, 177 |
|  | 3 | 1.296296 | 1.296286 | 1.296282 | 1.296280 | 92, 105, 238 | 95, 108, 204 | 138, 193, 250 | 99, 104, 212 |
|  | 4 | 1.654392 | 1.654390 | 1.654382 | 1.654320 | 35, 121, 170, 199 | 18, 118, 168, 189 | 14, 116, 134, 163 | 55, 107, 168, 231 |
|  | 5 | 1.999993 | 1.999991 | 1.999984 | 1.999900 | 83, 103, 169, 236, 256 | 68, 156, 157, 225, 255 | 33, 76, 110, 111, 189 | 69, 74, 107, 132, 197 |
| $2(512 \times 512)$ | 2 | 0.899171 | 0.898077 | 0.897812 | 0.892253 | 68, 204 | 66, 199 | 77, 212 | 66, 99 |
|  | 3 | 1.299174 | 1.297668 | 1.295387 | 1.287382 | 52, 138, 212 | 67, 130, 198 | 146, 175, 240 | 12, 113, 179, |
|  | 4 | 1.659995 | 1.659987 | 1.655493 | 1.656630 | 46, 104, 161, 203 | 30, 105, 162, 189 | 75, 115, 164, 199 | 51, 78, 150, 179 |
|  | 5 | 1.999997 | 1.999955 | 1.999982 | 1.999985 | 56, 89, 153, 195, 226 | 36, 90, 139, 186, 219 | 12, 24, 96, 138, 140 | 69, 74, 107, 132, 197 |
| $3(616 \times 616)$ | 2 | 0.888930 | 0.888239 | 0.885786 | 0.882399 | 61, 159 | 70, 130 | 14, 137 | 80, 108 |
|  | 3 | 1.296299 | 1.296290 | 1.296192 | 1.294283 | 69, 180, 201 | 78, 177, 215 | 8, 167, 255 | 89, 185, 216 |
|  | 4 | 1.659990 | 1.659991 | 1.654419 | 1.654222 | 56, 66, 109, 197 | 50, 63, 111, 204 | 18, 135, 189, 204 | 52, 65, 94, 178 |
|  | 5 | 1.999991 | 1.999990 | 1.999979 | 1.999919 | 65, 66, 127, 145, 184 | 70, 129, 192, 241, 255 | 6, 36, 99, 146, 186 | 08, 35, 154, 227, 240 |
| $4(512 \times 512)$ | 2 | 0.888892 | 0.888862 | 0.888465 | 0.888206 | 66, 178 | 62, 178 | 49, 174 | 74, 125 |
|  | 3 | 1.299488 | 1.299077 | 1.297182 | 1.297481 | 48, 138, 202 | 52, 142, 199 | 62, 122, 197 | 34, 202, 226 |
|  | 4 | 1.659922 | 1.650984 | 1.655421 | 1.655544 | 75, 144, 180, 211 | 72, 140, 162, 199 | 136, 146, 161, 222 | 161, 164, 183, 186 |
|  | 5 | 1.999995 | 1.999994 | 1.999994 | 1.998369 | 13, 64, 139, 160, 182 | 108, 171, 229, 254, 255 | 13, 22, 86, 170, 208 | 26, 117, 188, 223, 231 |
| $5(900 \times 900)$ | 2 | 0.888889 | 0.888886 | 0.888889 | 0.888806 | 69, 178 | 69, 178 | 129, 226 | 69, 176 |
|  | 3 | 1.299346 | 1.999922 | 1.296335 | 1.292477 | 52, 82, 199 | 52, 80, 190 | 7, 55, 166 | 51, 100, 225 |
|  | 4 | 1.659233 | 1.659055 | 1.656620 | 1.659716 | 64, 99, 104, 189 | 62, 92, 105, 195 | 63, 92, 98, 175 | 114, 196, 221, 246 |
|  | 5 | 1.999999 | 1.999996 | 1.999996 | 1.999897 | 10, 33, 121, 196, 287 | 96, 151, 221, 222, 255 | 60, 60, 98, 157, 199 | 59, 80, 87, 146, 231 |

Table 5 Comparison of PSNR (dB) and MSE values between BFHS, BF, HS and GA based technique using Kapur's entropy

| Test images | $m$ | PSNR (dB) |  |  |  | MSE |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  | BFHS | BF | HS | GA | BFHS | BF | HS | GA |
| $1(750 \times 750)$ | 2 | 24.640759 | 24.659402 | 24.649714 | 24.691119 | 2.233595 | 2.224027 | 2.228993 | 2.207844 |
|  | 3 | 24.838077 | 24.867909 | 24.840048 | 24.904497 | 2.134384 | 2.119773 | 2.133415 | 2.101989 |
|  | 4 | 25.065555 | 25.039729 | 25.063891 | 25.978840 | 2.025465 | 2.037546 | 2.026241 | 1.641331 |
|  | 5 | 26.219373 | 26.133128 | 25.241718 | 25.859903 | 1.426084 | 1.584044 | 1.944950 | 1.686902 |
| $2(512 \times 512)$ | 2 | 24.545122 | 24.542932 | 24.551459 | 24.582743 | 2.283327 | 2.284478 | 2.279997 | 2.263632 |
|  | 3 | 24.765263 | 24.792172 | 24.747597 | 24.756188 | 2.170471 | 2.157064 | 2.179318 | 2.175011 |
|  | 4 | 24.976139 | 25.012000 | 24.969124 | 25.897146 | 1.006857 | 2.050597 | 2.070941 | 1.672498 |
|  | 5 | 25.826587 | 25.760391 | 25.188178 | 25.676672 | 1.240895 | 1.726001 | 1.969076 | 1.759596 |
| $3(616 \times 616)$ | 2 | 24.626587 | 24.606034 | 24.575999 | 24.591409 | 2.240895 | 2.251525 | 2.26715 | 2.259120 |
|  | 3 | 24.801272 | 24.801903 | 24.840962 | 24.811471 | 2.152549 | 2.152236 | 2.132967 | 2.147500 |
|  | 4 | 25.024089 | 25.008767 | 25.008374 | 25.979138 | 2.044896 | 2.033309 | 2.052309 | 1.641218 |
|  | 5 | 25.833679 | 25.542806 | 25.238145 | 26.106857 | 1.697119 | 1.814678 | 1.946551 | 1.593656 |
| $4(512 \times 512)$ | 2 | 24.537128 | 24.569481 | 24.701880 | 24.697130 | 2.287533 | 2.270555 | 2.202380 | 2.204790 |
|  | 3 | 24.808983 | 24.784945 | 24.790649 | 24.903379 | 2.14873 | 2.160657 | 2.15782 | 2.102530 |
|  | 4 | 25.101653 | 25.052971 | 25.008708 | 25.051311 | 2.008699 | 2.031342 | 2.052152 | 1.690243 |
|  | 5 | 25.916055 | 25.627852 | 25.234411 | 25.853588 | 1.665232 | 1.779488 | 1.948225 | 1.650902 |
| $5(900 \times 900)$ | 2 | 24.320651 | 24.513275 | 24.467392 | 24.484757 | 2.404447 | 2.300132 | 2.324561 | 2.315285 |
|  | 3 | 24.659718 | 24.630789 | 24.703762 | 24.709665 | 2.223865 | 2.238728 | 2.201425 | 2.198436 |
|  | 4 | 24.860823 | 24.847207 | 24.954384 | 25.893057 | 2.123234 | 2.129901 | 2.077982 | 1.674073 |
|  | 5 | 26.066495 | 25.957263 | 25.206009 | 26.150378 | 1.028536 | 1.076605 | 1.961008 | 1.577765 |

Table 6 Comparison of CPU Timing, SSIM and FSIM between BFHS, BF, HS and GA based technique using Kapur's entropy

| Test images | $m$ | CPU timing |  |  |  | SSIM |  |  |  | FSIM |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  | BFHS | BF | HS | GA | BFHS | BF | HS | GA | BFHS | BF | HS | GA |
| $1(750 \times 750)$ | 2 | 2.81911 | 8.235545 | 25.22988 | 436.11369 | 0.918842 | 0.902112 | 0.943819 | 0.947394 | 0.772895 | 0.722617 | 0.895365 | 0.897415 |
|  | 3 | 3.286868 | 8.075897 | 24.80887 | 433.87366 | 0.961164 | 0.959592 | 0.968163 | 0.963107 | 0.884754 | 0.882816 | 0.937511 | 0.928888 |
|  | 4 | 2.612452 | 6.917417 | 24.86667 | 438.91758 | 0.978343 | 0.961763 | 0.979184 | 0.976280 | 0.947416 | 0.900145 | 0.955266 | 0.909506 |
|  | 5 | 2.821316 | 8.247721 | 25.73357 | 296.15088 | 0.989280 | 0.977779 | 0.985351 | 0.981165 | 0.964843 | 0.939080 | 0.964817 | 0.955579 |
| $2(512 \times 512)$ | 2 | 2.843121 | 6.890611 | 21.00694 | 185.94096 | 0.900747 | 0.865230 | 0.936475 | 0.941127 | 0.669172 | 0.579772 | 0.894690 | 0.891273 |
|  | 3 | 2.48574 | 6.641995 | 22.24080 | 181.80301 | 0.966901 | 0.953207 | 0.965857 | 0.962966 | 0.914892 | 0.806251 | 0.937512 | 0.936411 |
|  | 4 | 2.528806 | 6.336246 | 21.84823 | 184.04964 | 0.97696 | 0.970332 | 0.977355 | 0.969124 | 0.935541 | 0.906891 | 0.955900 | 0.897320 |
|  | 5 | 3.181302 | 6.661542 | 23.07188 | 121.46275 | 0.989512 | 0.981282 | 0.984123 | 0.979080 | 0.967901 | 0.942435 | 0.966675 | 0.944366 |
| $3(616 \times 616)$ | 2 | 2.947659 | 6.510813 | 24.00778 | 268.01849 | 0.886746 | 0.877771 | 0.945456 | 0.945627 | 0.711995 | 0.690732 | 0.910538 | 0.909899 |
|  | 3 | 2.487444 | 6.295622 | 23.27194 | 266.77671 | 0.958891 | 0.949243 | 0.967704 | 0.965602 | 0.884364 | 0.861097 | 0.946963 | 0.950250 |
|  | 4 | 2.621718 | 7.009322 | 23.85787 | 268.79580 | 0.976986 | 0.974446 | 0.979841 | 0.975580 | 0.947437 | 0.927941 | 0.967487 | 0.910598 |
|  | 5 | 2.733291 | 6.570533 | 24.95336 | 183.32426 | 0.985457 | 0.978247 | 0.985389 | 0.982706 | 0.978693 | 0.957271 | 0.977198 | 0.959924 |
| $4(512 \times 512)$ | 2 | 2.881887 | 6.883393 | 21.67048 | 184.39762 | 0.919911 | 0.916411 | 0.942495 | 0.941113 | 0.808156 | 0.758135 | 0.924892 | 0.924158 |
|  | 3 | 2.901766 | 6.518426 | 22.29530 | 180.80266 | 0.965901 | 0.939357 | 0.967192 | 0.958473 | 0.920468 | 0.809099 | 0.962968 | 0.969056 |
|  | 4 | 2.583831 | 6.224254 | 23.29745 | 185.24697 | 0.978174 | 0.977754 | 0.978779 | 0.973418 | 0.958493 | 0.954723 | 0.975200 | 0.946636 |
|  | 5 | 2.670841 | 6.753141 | 22.89839 | 123.92201 | 0.985296 | 0.979699 | 0.984767 | 0.981415 | 0.984812 | 0.965946 | 0.982297 | 0.963563 |
| $5(900 \times 900)$ | 2 | 2.706739 | 3.260656 | 25.04838 | 609.70270 | 0.888275 | 0.869001 | 0.912923 | 0.931914 | 0.763680 | 0.61448 | 0.935430 | 0.910566 |
|  | 3 | 3.18756 | 3.141117 | 26.31972 | 611.34520 | 0.947652 | 0.924667 | 0.952287 | 0.961039 | 0.854385 | 0.841511 | 0.962090 | 0.936919 |
|  | 4 | 3.782609 | 3.073785 | 26.74656 | 594.61794 | 0.96061 | 0.948236 | 0.972788 | 0.972305 | 0.941277 | 0.805258 | 0.975354 | 0.922978 |
|  | 5 | 3.317179 | 3.118346 | 26.48113 | 400.37624 | 0.982745 | 0.969831 | 0.981889 | 0.977216 | 0.989692 | 0.939845 | 0.982301 | 0.949224 |

Table 7 Comparison of best objective function values and their corresponding threshold values between BFHS, BF, HS and GA based technique using Otsu

| Test images | $m$ | Best objective function values |  |  |  | Optimum threshold values |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  | BFHS | BF | HS | GA | BFHS | BF | HS | GA |
| $1(750 \times 750)$ | 2 | 0.9324672 | 0.9217461 | 0.9018988 | 0.8962822 | 92, 165 | 90,165 | 90, 164 | 89, 159 |
|  | 3 | 0.9557924 | 0.9429610 | 0.9416061 | 0.9160700 | 50, 130, 203 | 50, 118, 201 | 50, 116, 181 | 50, 136, 201 |
|  | 4 | 0.9685061 | 0.9510638 | 0.9636437 | 0.9558676 | 60, 90, 164, 195 | 62, 90, 160, 185 | 29, 87, 134, 194 | 69, 108, 184, 184 |
|  | 5 | 0.9953252 | 0.9897911 | 0.9835491 | 0.9738400 | 32, 69, 123, 174, 224 | 61, 118, 197, 239, 254 | 59, 91, 133, 179, 217 | 41, 128, 215, 221, 247 |
| $2(512 \times 512)$ | 2 | 0.9379702 | 0.9375483 | 0.9235074 | 0.9205700 | 92, 176 | 90, 170 | 92, 173 | 88, 170 |
|  | 3 | 0.9510369 | 0.9503637 | 0.9480333 | 0.9419880 | 65, 148, 201 | 69, 130, 201 | 62, 138,204 | 70, 120, 185 |
|  | 4 | 0.9780577 | 0.9668738 | 0.9661590 | 0.9552768 | 52, 99, 132, 190 | 58, 92, 132, 188 | 56, 95, 130, 179 | 109, 162, 217, 242 |
|  | 5 | 0.998046 | 0.9966540 | 0.9947617 | 0.9717720 | 35, 81, 127, 175, 217 | 58, 106, 141, 173, 227 | 49, 85, 132, 171, 213 | 41, 88, 125, 209, 249 |
| $3(616 \times 616)$ | 2 | 0.9205902 | 0.9053900 | 0.9231815 | 0.9155424 | 90, 177 | 92, 164 | 91, 168 | 77, 164 |
|  | 3 | 0.9550135 | 0.9514701 | 0.9496108 | 0.944740 | 65, 142, 195 | 62, 138, 188 | 65, 133, 193 | 65, 141, 184 |
|  | 4 | 0.9737103 | 0.9687232 | 0.9670056 | 0.9587931 | 60, 90, 169, 214 | 52, 96, 160, 203 | 45, 94, 159, 200 | 88, 161, 208, 245 |
|  | 5 | 0.989655 | 0.9818421 | 0.9805966 | 0.9834300 | 24, 70, 117, 169, 222 | 40, 86, 146, 200, 238 | 66, 103, 148, 171, 218 | 23, 50, 162, 195, 248 |
| $4(512 \times 512)$ | 2 | 0.9218636 | 0.9172908 | 0.9087261 | 0.9038803 | 77, 190 | 72, 188 | 73, 171 | 104, 174 |
|  | 3 | 0.9489847 | 0.9307470 | 0.9318175 | 0.9253098 | 60, 118, 189 | 68, 152, 177 | 62, 108, 183 | 88, 158, 199 |
|  | 4 | 0.9748262 | 0.9480545 | 0.9557225 | 0.9517529 | 52, 98, 149, 204 | 36, 98, 147, 210 | 51, 97, 147, 210 | 34, 98, 171, 171 |
|  | 5 | 0.9974924 | 0.9881785 | 0.9768757 | 0.9660907 | 47, 95, 145, 187, 227 | 68, 104, 155, 211, 254 | 61, 94, 128, 180, 199 | 27, 62, 151, 204, 245 |
| $5(900 \times 900)$ | 2 | 0.9128477 | 0.9108475 | 0.9092101 | 0.8993944 | 80, 170 | 82, 172 | 87, 176 | 97, 172 |
|  | 3 | 0.955479 | 0.9376586 | 0.9256240 | 0.9106151 | 88, 138, 204 | 86, 135, 204 | 82, 134, 204 | 72, 122, 201 |
|  | 4 | 0.9618102 | 0.9570892 | 0.9439054 | 0.9542124 | 64, 123, 168, 214 | 62, 113, 160,214 | 64, 119, 160, 212 | 128, 207, 221 |
|  | 5 | 0.9979752 | 0.9849869 | 0.9803246 | 0.9843358 | 39, 78, 120, 166, 210 | 30, 59, 113, 164, 215 | 50, 79, 134, 155, 185 | 48, 123, 162, 207, 245 |

Table 8 Comparison of PSNR (dB) and MSE values between BFHS, BF, HS, and GA-based technique using Otsu

| Test images | $m$ | PSNR (dB) |  |  |  | MSE |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  | BFHS | BF | HS | GA | BFHS | BF | HS | GA |
| $1(750 \times 750)$ | 2 | 24.653841 | 24.648178 | 24.644845 | 24.645697 | 2.226877 | 2.229782 | 2.2314943 | 2.231056 |
|  | 3 | 24.856153 | 24.870026 | 24.859707 | 24.870600 | 2.125519 | 2.118740 | 2.1237805 | 211.8451 |
|  | 4 | 25.067745 | 25.054861 | 25.066102 | 26.279928 | 2.024444 | 2.030459 | 2.0252100 | 1.531396 |
|  | 5 | 25.769200 | 25.082472 | 25.293798 | 25.124000 | 1.722504 | 2.017590 | 1.9217658 | 199.8379 |
| $2(512 \times 512)$ | 2 | 24.56352 | 24.579735 | 24.546290 | 24.552551 | 2.273674 | 2.265201 | 2.2827130 | 2.279424 |
|  | 3 | 24.780481 | 24.759637 | 24.732067 | 24.764200 | 2.162878 | 2.173284 | 2.1871251 | 217.1000 |
|  | 4 | 24.981269 | 25.006653 | 25.012021 | 24.874078 | 2.065158 | 2.053123 | 2.0505869 | 2.116764 |
|  | 5 | 25.627989 | 25.4729200 | 25.197580 | 25.004700 | 1.779431 | 1.844116 | 1.9648177 | 205.4032 |
| $3(616 \times 616)$ | 2 | 24.621500 | 24.583300 | 24.585789 | 24.570418 | 2.24352 | 2.263334 | 2.2620458 | 2.270065 |
|  | 3 | 24.800300 | 24.794900 | 24.836735 | 24.786600 | 2.153032 | 2.155715 | 2.1350436 | 215.9814 |
|  | 4 | 24.968400 | 25.004600 | 24.973612 | 24.974895 | 2.071303 | 2.054118 | 2.0688030 | 2.068191 |
|  | 5 | 25.771400 | 25.300500 | 25.234673 | 25.158200 | 1.721626 | 1.918794 | 1.9481078 | 198.2723 |
| $4(512 \times 512)$ | 2 | 24.652600 | 24.596500 | 24.683958 | 24.612642 | 2.227535 | 2.256463 | 2.2114878 | 2.248102 |
|  | 3 | 24.890500 | 24.800200 | 24.826838 | 24.828800 | 2.108757 | 2.153061 | 2.1399148 | 213.8947 |
|  | 4 | 25.100800 | 25.014100 | 24.998579 | 26.075901 | 2.009097 | 2.049629 | 2.0569436 | 1.605056 |
|  | 5 | 25.397300 | 25.136500 | 25.323115 | 25.007600 | 1.876502 | 1.992666 | 1.9088367 | 205.2699 |
| $5(900 \times 900)$ | 2 | 24.528200 | 24.518400 | 24.477724 | 24.461037 | 2.292235 | 2.297417 | 2.3190380 | 2.327965 |
|  | 3 | 24.70400 | 24.705900 | 24.712801 | 24.725000 | 2.201311 | 2.200361 | 2.1968492 | 219.0674 |
|  | 4 | 24.935600 | 24.953200 | 24.948091 | 24.958259 | 2.087012 | 2.078549 | 2.0809957 | 2.076129 |
|  | 5 | 25.792700 | 25.680600 | 25.170055 | 25.013500 | 1.713199 | 1.757999 | 1.9773103 | 204.9903 |

Table 9 Comparison of CPU Timing, SSIM, and FSIM between BFHS, BF, HS and GA based technique using Otsu

| Test images | $m$ | CPU timing |  |  |  | SSIM |  |  |  | FSIM |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  | BFHS | BF | HS | GA | BFHS | BF | HS | GA | BFHS | BF | HS | GA |
| $1(750 \times 750)$ | 2 | 4.667568 | 5.007778 | 7.29691 | 272.89039 | 0.920762 | 0.904131 | 0.9423958 | 0.942725 | 0.772803 | 0.728028 | 0.8925695 | 0.889432 |
|  | 3 | 5.363554 | 5.276051 | 6.587852 | 275.03171 | 0.963812 | 0.968801 | 0.9687778 | 0.962500 | 0.917134 | 0.929694 | 0.9360198 | 0.924800 |
|  | 4 | 5.076405 | 5.519536 | 7.154403 | 272.78504 | 0.971162 | 0.972597 | 0.9778467 | 0.955294 | 0.936363 | 0.938145 | 0.9487690 | 0.870566 |
|  | 5 | 4.784902 | 5.427858 | 6.753919 | 275.44405 | 0.982051 | 0.967690 | 0.9819275 | 0.95570000 | 0.961703 | 0.922072 | 0.9597429 | 0.894800 |
| $2(512 \times 512)$ | 2 | 2.251826 | 2.511814 | 13.23507 | 121.92735 | 0.907199 | 0.898708 | 0.9341685 | 0.936426 | 0.690586 | 0.673149 | 0.8938984 | 0.892676 |
|  | 3 | 2.383504 | 2.495365 | 4.30062 | 118.78825 | 0.958494 | 0.950182 | 0.9613643 | 0.964400 | 0.834441 | 0.803622 | 0.9362953 | 0.928500 |
|  | 4 | 2.243267 | 2.596011 | 4.000213 | 119.73264 | 0.97805 | 0.976883 | 0.9771952 | 0.924582 | 0.929875 | 0.912289 | 0.9267295 | 0.892821 |
|  | 5 | 2.228959 | 2.434094 | 4.336584 | 119.81306 | 0.984679 | 0.978082 | 0.9837941 | 0.972000 | 0.968999 | 0.942692 | 0.9662359 | 0.929000 |
| $3(616 \times 616)$ | 2 | 3.244759 | 3.504219 | 5.748055 | 174.33693 | 0.885600 | 0.869500 | 0.9450091 | 0.945003 | 0.704500 | 0.668900 | 0.9009851 | 0.903914 |
|  | 3 | 3.437252 | 3.588553 | 5.722739 | 174.18969 | 0.958300 | 0.957800 | 0.9666987 | 0.962600 | 0.881700 | 0.885600 | 0.9464230 | 0.929000 |
|  | 4 | 3.440058 | 3.744837 | 5.670794 | 174.57630 | 0.978500 | 0.978500 | 0.9784969 | 0.960120 | 0.971100 | 0.967400 | 0.9639295 | 0.939038 |
|  | 5 | 3.406708 | 3.732528 | 5.793264 | 173.69577 | 0.981600 | 0.979800 | 0.9806148 | 0.947300 | 0.968800 | 0.966400 | 0.9680752 | 0.929800 |
| $4(512 \times 512)$ | 2 | 2.122679 | 2.358085 | 4.154642 | 120.80439 | 0.922700 | 0.917100 | 0.9384357 | 0.920310 | 0.767700 | 0.750400 | 0.9305519 | 0.910021 |
|  | 3 | 2.194987 | 2.365144 | 4.415085 | 117.20405 | 0.965300 | 0.96400 | 0.9668690 | 0.947000 | 0.899300 | 0.914800 | 0.9562953 | 0.9399 |
|  | 4 | 2.345375 | 2.634723 | 4.647286 | 122.83085 | 0.978700 | 0.977700 | 0.9779908 | 0.944344 | 0.958200 | 0.975200 | 0.9782707 | 0.915081 |
|  | 5 | 2.366824 | 2.664677 | 4.727931 | 118.01789 | 0.982700 | 0.975600 | 0.9817766 | 0.961100 | 0.972300 | 0.969900 | 0.9680768 | 0.951300 |
| $5(900 \times 900)$ | 2 | 8.237369 | 8.095456 | 8.227432 | 401.23638 | 0.894200 | 0.905700 | 0.9361244 | 0.931305 | 0.681000 | 0.710500 | 0.9231666 | 0.905986 |
|  | 3 | 7.747557 | 7.904731 | 8.647928 | 398.72867 | 0.960500 | 0.958400 | 0.9527245 | 0.956800 | 0.872100 | 0.862200 | 0.9445500 | 0.949800 |
|  | 4 | 7.998608 | 8.594646 | 8.813292 | 403.10783 | 0.979300 | 0.978100 | 0.9712128 | 0.890741 | 0.955700 | 0.966800 | 0.9695881 | 0.838519 |
|  | 5 | 8.233814 | 8.289011 | 8.960079 | 404.39053 | 0.981400 | 0.977300 | 0.9801409 | 0.971100 | 0.956100 | 0.952200 | 0.9506624 | 0.952100 |

Results of $\mathbf{1}^{\text {st }}$ Test Image using Between-Class Variance


Fig. 8 Results of 1 st test satellite image using BFHS, BF, HS and GA with Otsu entropy. (a-d) 2-level to 5-level thresholding based segmented image with the best thresholds obtained from BFHS algorithm using Otsu's entropy criterion, (e-h) 2-level to 5-level thresholding based segmented image with the best thresholds obtained from BF algorithm using Otsu's entropy criterion, (i-l) 2-level to 5-level thresholding based segmented image with the best thresholds obtained from HS algorithm using Otsu's entropy criterion and ( $\mathbf{m}-\mathbf{p}$ ) 2-level to 5-level thresholding based segmented image with the best thresholds obtained from GA using Otsu's entropy criterion

In the BFHS case, the LA algorithm does not require initialization as it works with random initial values; however, in order to assure a valid comparison, the same initial values are considered for the BF, the HS, and the BFHS methods. Figure 14 shows a clear pixel misclassification in some sections of the image as a consequence of such sensitivity.


Fig. 9 Results of 2nd test satellite image using BFHS (a-d) 2-level to 5-level thresholding based segmented image with the best thresholds obtained from BFHS algorithm using Kapur's entropy criterion, (a'd') 2-level to 5-level thresholding based segmented image with the best thresholds obtained from BFHS algorithm using Otsu's entropy criterion

The experiment aims to measure the number of required steps and the computing time spent by the Expectation-Maximization (EM) [32], the (Levenberg-Marquardt) LM [30] and the LA algorithm needed to calculate the parameters of the Kapur and Otsu in satellite images. All experiments consider four threshold levels ( $m=2-5$ ). Table 10 shows the averaged measurements as they are obtained from 20 experiments.


Fig. 10 Results of $3 r d$ test satellite image using BFHS (a-d) 2-level to 5-level thresholding based segmented image with the best thresholds obtained from BFHS algorithm using Kapur's entropy criterion, ( $\mathrm{a}^{\prime}-\mathrm{d}^{\prime}$ ) 2-level to 5-level thresholding based segmented image with the best thresholds obtained from BFHS algorithm using Otsu's entropy criterion


Fig. 11 Results of 4th test satellite image using BFHS (a-d) 2-level to 5-level thresholding based segmented image with the best thresholds obtained from BFHS algorithm using Kapur's entropy criterion, (a'd') 2-level to 5-level thresholding based segmented image with the best thresholds obtained from BFHS algorithm using Otsu's entropy criterion

It is evident that the EM is the slowest to converge (iterations), and the LM shows the highest computational cost (elapsed time) because it requires complex Hessian approximations. On the other hand, the LA shows an acceptable compromise between its convergence time and its computational cost.


Fig. 12 Results of 5th test satellite image using BFHS (a-d) 2-level to 5-level thresholding based segmented image with the best thresholds obtained from BFHS algorithm using Kapur's entropy criterion, ( $a^{\prime}-d^{\prime}$ ) 2-level to 5-level thresholding based segmented image with the best thresholds obtained from BFHS algorithm using Otsu's entropy criterion

Fig. 13 Convergence curves of $\mathrm{BF}, \mathrm{HS}$, and BFHS algorithms for different threshold levels





Initial condition set number 1


Initial condition set number 2


HS


Fig. 14 Segmented images after applying the BF, the HS, and the BFHS algorithms with different initial conditions

### 5.4 Time complexity

The time complexity of BFHS is based on the complex nature of its inline processes, namely, the LA-based switching, calculating the quality of the segment, applying BF, and applying HS. Assuming $\mathrm{T}_{1}$ is the number of iterations taken by the BF to converge, and $\mathrm{T}_{2}$ is the number of iterations taken by the HS to converge. Then the complexity of LA-based switching is $\mathrm{O}\left(s T_{1} T_{2} N_{c}\right.$ $N_{p} N_{d}$ ), while the complexity of calculating the quality of a partition will depend on the time complexity of the validity index which is some constant, $q$, multiplied by $N_{p}$ for all the indices used in this paper. Therefore, the complexity of this step will be $\mathrm{O}\left(q T_{1} T_{2} N_{p}\right)$. The parameters $T_{1}, T_{2}, N_{c}$, $s$, and $\xi$ can be fixed in advance. Typically, $T_{1}, T_{2}, N_{c} s, \xi, N_{d} \ll N_{p}$. Let $\alpha$ be the multiplication of $T_{1}$, $T_{2}, N_{c}, s, N_{d}\left(\alpha=s \times T_{1} \times T_{2} \times N_{c} \times N_{d}\right)$. If $\alpha \leq N_{p}$ then the time complexity of BFHS will be $\mathrm{O}\left(N_{p}\right)$. However, if $\alpha \approx N_{p}$, then the time complexity of BFHS will be $\mathrm{O}\left(N_{p}{ }^{2}\right)$.

Table 10 Average iterations and time requirements of the EM, the LM, and the LA algorithm

| Iterations <br> Elapsed time | Image 1 | Image 2 | Image 3 | Image 4 | Image 5 |
| :--- | :--- | :--- | :--- | :--- | :--- |
| EM | 1855 | 1833 | 1861 | 1870 | 1925 |
|  | 7.06 s | 4.42 s | 5.58 s | 5.71 s | 10.23 s |
| LM | 985 | 988 | 945 | 958 | 1028 |
|  | 10.48 s | 6.62 s | 10.19 s | 10.43 s | 16.96 s |
| LA | 970 | 991 | 951 | 951 | 1009 |
|  | 3.92 s | 2.51 s | 3.03 s | 3.1 s | 5.65 s |

## 6 Conclusion

In this paper, an automatic image multi-threshold approach based on the combination of Bacterial Foraging (BF) optimization algorithm, Harmony Search (HS) algorithm and Learning Automata (LA) is proposed. In this approach, the initial population is produced by combining chaotic systems with opposition-based learning routine to improve global convergence rate. In this paper, BFHS, BF, HS and GA algorithms were exploited to maximize Kapur's entropy and between-class variance (Otsu) separately to find optimum multilevel thresholds. The results suggest that the hybrid algorithm (BFHS) with Kapur's and Otsu's entropy criterion can be efficiently used in multilevel thresholding segmentation for two set of images, namely standard and satellite image.

The numerical illustrations and fidelity assessments for almost every sample images, considered in this paper, demonstrate that the BFHS-Kapur's and BFHS-Otsu's outperform other competitive algorithms with each objective functions. The best part of BFHS is its computational efficiency, the accuracy of segmentation. Moreover, experimental evidence shows that LA algorithm has an acceptable compromise between its convergence time and its computational cost when it is compared to the Expectation-Maximization (EM) method and the Levenberg-Marquardt (LM) algorithm. The results have shown that the stochastic search accomplished by the LA method shows a consistent performance with no regard to the initial value and still indicating a greater chance to reach the global minimum.

The study also explores the comparison between the two objective functions with each optimization technique, which reveals that Kapur's entropy is superior for each sample images. The advantage of Kapur's entropy is that it uses a global and objective property of the histogram; because of its general nature, this criterion can be used for segmentation purpose. The validity and accuracy of the proposed BFHS based multilevel thresholding technique are reported both qualitatively and quantitatively. To measure the performance of proposed approach, uniformity, best objective value, MSE, SD, FSIM, SSIM, and PSNR, which assesses the segmentation quality, considering the coincidences between the segmented and the original images, has been used.

It is concluded that the order of CPU runtime from low to high are sorted in the order of $\mathrm{BFHS}<\mathrm{HS}<\mathrm{BF}<\mathrm{GA}$. This is due to the fast convergence rate of modified search initialization that the input parameters such as the number of iteration, the number of population size used are lesser than BF, HS, and GA.

The experimental results are very promising and encourage future research for applying BFHS to complex image processing application such as satellite image enhancement, satellite image denoising, and optimization based image classification and also in various computer vision problems. The performance of few more objective functions such as minimum cross entropy and Renyi's entropy can also be estimated using BFHS techniques for a standard set of images as well as for satellite images to check the robustness of this algorithm for multilevel thresholding problem.

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